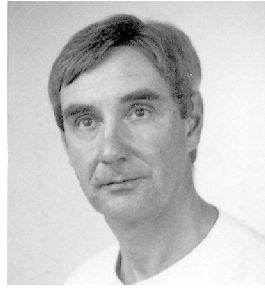


## REMARKS ON THE ELECTRON STRUCTURE FUNCTION

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## Abstract

The electron and photon structure functions are compared. Advantages of the electron structure function are demonstrated. At very high momenta probabilistic (partonic) interpretation can be preserved despite strong  $\gamma$ - $Z$  interference. At present energies analyses of both the electron and the photon structure functions give an important test of the experimentally applied methods. Predictions for the electron structure function at present and future momenta are given.

Let me start with the well known measurement of the photon structure function, displayed graphically in Fig. 1a. The tagged (upper) electron emits a probing photon whereas the untagged (lower) one goes nearly along the beam, emitting the target photon. The large scale  $Q^2 = -q^2$ , is determined solely by the tagged electron. The anti-tagging condition (if present) requires the virtuality of the target photon  $P_\gamma^2$  to be between its kinematically allowed minimum and certain fixed maximum  $P^2$ :

$$-(p - p')^2 \equiv P_\gamma^2 \leq P^2. \quad (1)$$

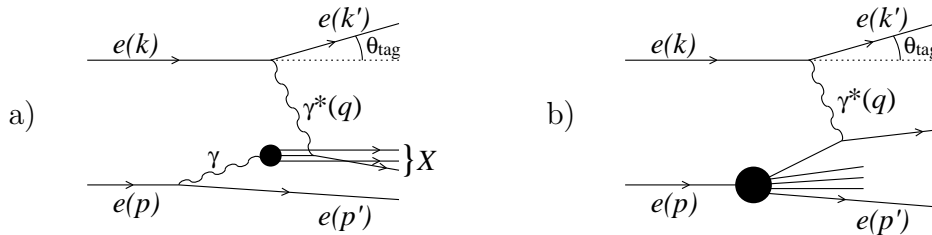


Figure 1: Deep inelastic scattering on a photon (a) and electron (b) target

In practice the average photon virtuality is very close to zero and the usual interpretation says that we are measuring the real photon structure function. This photon is clearly not a beam particle and has the energy diffused according to the equivalent photon (Weizsäcker-Williams<sup>3</sup>) spectrum  $f_\gamma^e$ . The measured cross-section for the production of a hadronic system  $X$ , expressed in terms of the photon structure functions  $F_2^\gamma$  and  $F_L^\gamma$  reads:

$$\frac{d^3\sigma_{ee \rightarrow eX}}{dzdQ^2} = \frac{2\pi\alpha^2}{x^2Q^4} [(1 + (1 - y)^2)F_2^\gamma(x, Q^2, P^2) - y^2F_L^\gamma(x, Q^2, P^2)] f_\gamma^e(z/x, P^2) dx \quad (2)$$

where

$$y = 1 - (E_{\text{tag}}/E) \cos^2(\theta_{\text{tag}}/2) \quad (3)$$

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and  $x(z)$  are fractions of parton momentum with respect to the photon (electron). We assume here, as well as in Eq. (5) below, that the probing boson is photon only. The  $Z$  boson contribution will be included for the predictions at high  $Q^2$ .

Few remarks are important for further considerations. First, the splitting of the process into a distribution of photons inside electron  $f_\gamma^e$  and that of partons inside the photon  $F_2^\gamma$  is an approximation. Second, in order to fix  $x$ , one is forced to measure — in addition to the tagged electron — the hadronic momenta. In fact,

$$x = \frac{Q^2}{Q^2 + P_\gamma^2 + W^2} \approx \frac{Q^2}{Q^2 + W^2}, \quad (4)$$

where  $W$  is the invariant mass of the produced hadronic system  $X$ . Its determination is more difficult than of other (tagged electron) variables. The uncertainty in the determination of the  $x$  variable is the source of biggest errors in the analysis. The data are indirectly biased by theoretical assumptions.

Some of the above problems can be avoided when we introduce the structure function of the electron (Fig. 1b). To see how it works let us first write the cross-section (2), this time in terms of the electron structure functions  $F_2^e$  and  $F_L^e$ :

$$\frac{d^2\sigma_{ee \rightarrow eX}}{dzdQ^2} = \frac{2\pi\alpha^2}{zQ^4} [(1 + (1 - y)^2)F_2^e(z, Q^2, P^2) - y^2F_L^e(z, Q^2, P^2)]. \quad (5)$$

No unfolding procedure is necessary to obtain  $F_2^e$ , and its argument  $z$  — the parton momentum fraction with respect to the electron — is measured, as in the standard deep inelastic scattering, by means of the tagged electron variables only:

$$z = \frac{Q^2}{2pq} = \frac{\sin^2(\theta_{\text{tag}}/2)}{E/E_{\text{tag}} - \cos^2(\theta_{\text{tag}}/2)}. \quad (6)$$

There is no need *a priori* to reconstruct the hadronic mass  $W$  (In practice a lower limit cut on  $W$  is imposed due to difficulties in experimental reconstruction of very low hadronic masses.). All these features cause that the same experiment can produce more precise and analysis independent data when looking at the electron structure. What is most important — the electron structure function contains the same information about QCD as the photon one and is known theoretically with at least the same accuracy. Moreover, it allows to avoid problems which arise in the photon structure function at very high energies.

The construction of the QCD electron structure function can be summarized in two steps presented in detail in Refs. <sup>4,5</sup>. First we calculate the splitting function of the electron into a quark/anti-quark. In this process we allow, in addition to the photon, for the exchange of  $Z$  and  $W$  bosons even if at present energies the photon contribution dominates (this generalisation allows for the  $\gamma - Z$  interference). In the second step we construct the  $Q^2$ -evolution equations for the quark and gluon densities inside the electron  $q(z, t)$  and  $G(z, t)$ . I skip an interesting discussion of scales entering the process (see Refs. <sup>4,5</sup>). Let me only present the asymptotic solutions to the evolution equations in the case with fixed antitag momentum squared  $P^2$ . They take the form ( $t = \log(Q^2/\Lambda_{QCD}^2)$  and  $t_1 = \log(P^2/\Lambda_{QCD}^2)$ ):

$$q(z, t) \simeq \left(\frac{\alpha}{2\pi}\right)^2 q^{\text{as}}(z) t_1 t, \quad G(z, t) \simeq \left(\frac{\alpha}{2\pi}\right)^2 G^{\text{as}}(z) t_1 t \quad (7)$$

with  $q^{\text{as}}(z)$  and  $G^{\text{as}}(z)$  being given by known,  $t$ -independent integral equations <sup>4,5</sup>. Their numerical solutions are shown in Fig. 2. As expected in the asymptotic region all bosons contribute. What is interesting, the  $\gamma$ - $Z$  interference term enters with strength comparable to the contribution of the  $Z$  boson itself. This means that the notion of separate equivalent bosons breaks down at very high momenta. The electron structure function takes correctly into account interference effects, preserving at the same time probabilistic (partonic) interpretation. One should keep in mind that only in the asymptotic region contributions from all intermediate bosons entering the splitting function are comparable. At lower energies the photon contribution dominates.

Is it possible to measure the above mentioned new effects in the next generation of  $e^+e^-$  colliders? The answer is given in Fig. 3a where the electron structure function  $F_2^e$  is shown for CLIC <sup>6</sup> energies. In this figure we use our asymptotic solutions for the  $z$  dependence but we multiply each contribution by the appropriate, finite logarithm. At such high  $Q^2$  the electron is probed of course by both the photon and the  $Z$  boson. The effect of  $Z$  and  $\gamma$ - $Z$  interference terms in the structure function is of the order of 5 to 15%. Effects of even larger size can be observed in double-tag experiment <sup>7</sup>.

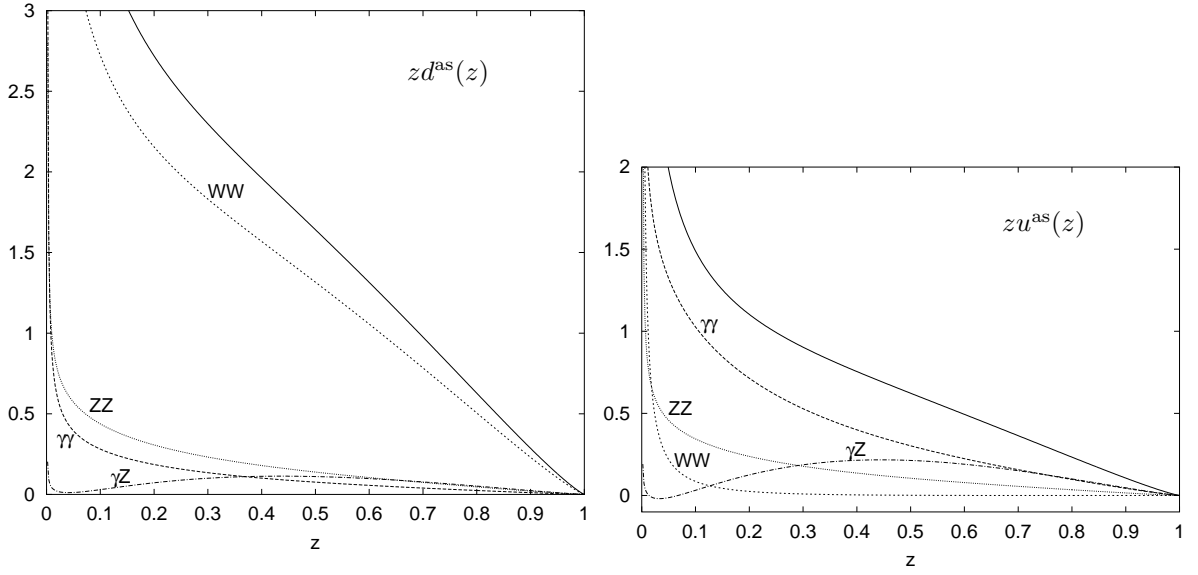


Figure 2: Asymptotic quark distributions — solid line. The other lines show different contributions as labeled

At present energies, where the  $W$  and  $Z$  contributions are negligible, one can reanalyze the existing data in terms of the electron structure function. As already mentioned this can be treated also as a consistency check of both photon and electron structure. Having a parametrization of the photon structure function which describes well the existing data we can predict the electron structure function by taking the convolution of this parametrization with the equivalent photon spectrum. The curves resulting from some popular parametrizations<sup>8</sup> are shown in Fig. 3b.

Let us add a few final remarks. First concerns the study of the virtual photon structure<sup>9</sup>. The analysis can be reformulated in terms of the  $P_\gamma^2$  dependence of the electron structure function. Studying a real, convention independent object is the first advantage. Another one is the fact that at very high virtualities the  $Z$  admixture and the  $\gamma$ - $Z$  interference are properly taken into account.

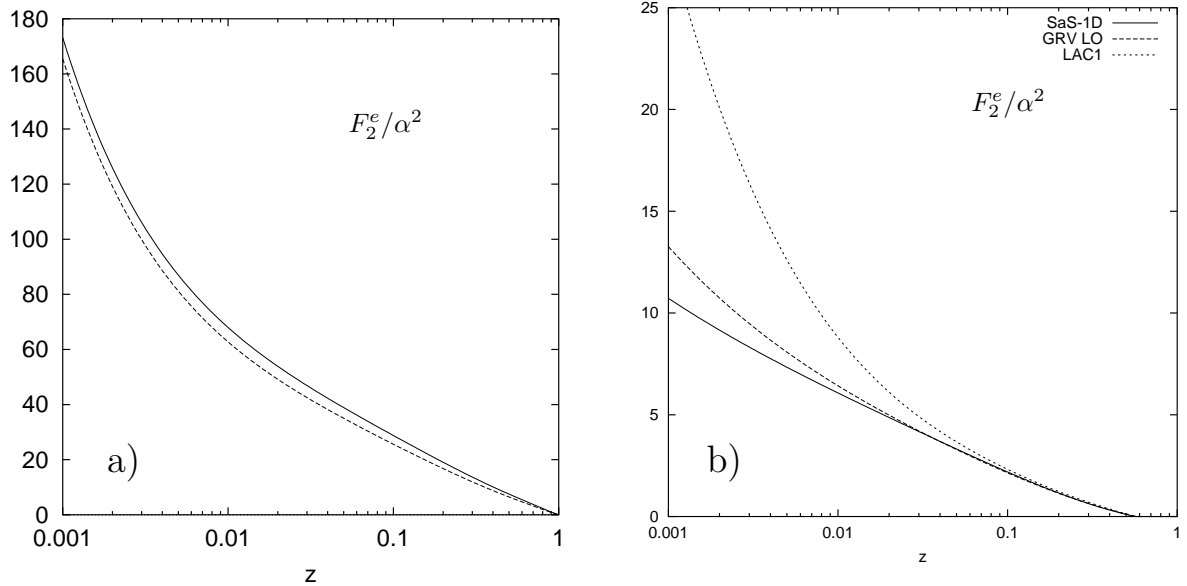


Figure 3: Predicted value of the electron structure function  $F_2^e(z)/\alpha^2$ : a) asymptotic solution at  $Q^2 = 50000 \text{ GeV}^2$  and  $P^2$  up to  $1000 \text{ GeV}^2$  (solid line), the contribution from the photon only is also shown (broken line); b) at  $Q^2 = 12 \text{ GeV}^2$  with the cut on the hadronic mass  $W > 3 \text{ GeV}$  from different parametrizations of the photon structure function: SaS (solid line), GRV (broken line), LAC1 (dotted line).

We also comment on the QED structure function of the photon. It is obtained from the process  $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$  by dividing out the (approximate) equivalent photon distribution and assuming some average photon virtuality. The use of the QED electron structure function avoids these problems. The exactly known (in given order of  $\alpha$ ) electron structure function can be compared directly with the electron data.

Finally, the photon structure function has been also measured<sup>10</sup> in di-jet production at HERA. Again the extraction of the  $x$  variable is difficult. In addition to jets, one has to measure essentially the whole hadronic system in order to obtain the photon energy. The data, when presented in terms of the electron structure function, require only measurement of the two jets.

To summarize, we propose to look at the electron as surrounded by a QCD cloud of quarks and gluons (in order  $\alpha^2$ ), very much like it is surrounded by a QED cloud of equivalent photons (in order  $\alpha$ ). We argue that the use of the electron structure function in electron induced processes has important advantages over the photon one. Experimentally it leads to more precise, convention independent data. Theoretically it allows for more careful treatment of all variables. It also takes into account all electroweak gauge boson contributions, including their interference, which will be important in the next generation of  $e^+e^-$  colliders. At present energies it should certainly be used as a cross-check of the photon structure analysis. In fact two experimental groups at CERN are currently performing the electron structure function analyses.

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